SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Fisher's Z-distribution.

17. Show that the sequence $\{F_n(t)\}$ of distribution functions of student's t with n degree of freedom satisfies for every t the relation

$$\lim_{n\to\infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-t^2/2}\, dt .$$

18. Derive the distribution of $\bar{x}$ for some non-normal population.

19. Prove that A_n unbiased estimates U of the parameter Q is the most efficient if and only if

(a) The estimate U is sufficient

(b) For $g(u, Q) > 0$, the density $g(u,Q)$ almost everywhere satisfies the relation. $\frac{\partial \log g(u, Q)}{\partial Q} = C(u - Q)$, where the number C is independent of U (if map depend on Q).

20. Derive the OC function of the sequential probability ratio test.

NOVEMBER/DECEMBER 2024

GEMA24A/DEMA24A — MATHEMATICAL STATISTICS

Time : Three hours Maximum : 75 marks

MAHARAJA ARTS & SCIENCE COLLEGE LIBRARY

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define statistic

2. Define sample

3. Define consistents estimates

4. Define unbiased estimates

5. Definition of sufficient estimate

6. Define asymptotically most efficient estimates

7. Define likelihood function

8. Define alternative hypotheses

9. Define sequential probability ratio test

10. Write the determination of A and B value.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) From a population in which the characteristics X has the normal distribution $N(1,2)$, we draw a simple sample of size $n=12$, we observe the following values of x:

$x_1=2.0, x_2=2.5, x_3=0.5, x_4=1.0$

$x_5=0.0, x_6=-0.9$

$x_7=5.1, x_8=-1.5, x_9=0.8,$

$x_{10}=1.1, x_{11}=0.8, x_{12}=0.4$

Or

(b) Verify the χ^2 test non parameter hypotheses.

12. (a) The characteristic X of elements of a population has the Poisson distribution $P_k=P(x=k)=\frac{\lambda^k}{k!}e^{-\lambda}, (k=0,1,...)$ where the parameter λ is unknown.

Or

(b) Derive the χ^2 test.

SRM INSTITUTE OF SCIENCE & TECHNOLOGY LIBRARY

13. (a) Let $S_n(x)$ be the empirical distribution function of a simple sample of size n drawn from a population in which the random variable X has a continuous distribution function $F(x)$, then

$\lim_{n\to\infty} Q_n(\lambda)=Q(\lambda)=$

$$\begin{cases} \sum_{k=-\infty}^{\infty}(-1)^k \exp(-2k^2\lambda^2), & \text{for } \lambda>0 \\ 0, & for\lambda\le 0 \end{cases}$$

Or

(b) Prove that : If U is the most efficient estimates of parameter Q, then almost everywhere in the domain where $g(u, Q)>0$, u is the solution of known equation.

14. (a) Show that the determination of A and B.

Or

(b) Testing a hypotheses concerning the parameter P of A zero-one distribution.

15. (a) Prove that, If the variance of Z is different from zero, the probabilities that $n=\infty$ is equal to zero.

Or

(b) Derive the expected value $E(n)$.